Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

ADDITIONAL MATHEMATICS **0606/23**

Paper 2 **October/November 2016**

MARK SCHEME

Maximum Mark: 80

Published

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Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied
www without wrong working

Question	Answer	Mark	Part Marks
1	$\dfrac{\left(\sqrt{5}+3\sqrt{3}\right)}{\left(\sqrt{5}+\sqrt{3}\right)}\times\dfrac{\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}-\sqrt{3}\right)}$	M1	rationalise with $\left(\sqrt{5}-\sqrt{3}\right)$
	$=\dfrac{5+3\sqrt{15}-\sqrt{15}-9}{5-3}$	A1	numerator (3 or 4 terms)
	$=\dfrac{2\sqrt{15}-4}{2}=\sqrt{15}-2$	A1	denominator and completion
2	$\ln e^{3x}=\ln 6e^{x}$		
	$3x=\ln 6e^{x}$	M1	one law of indices/logs
	$3x=\ln 6+\ln e^{x}$	M1	second law of indices/logs
	$3x=\ln 6+x$		
	$x=\dfrac{1}{2}\ln 6$ or $\ln\sqrt{6}$ or 0.896	A1	www oe in base 10
3 (i)	$\dfrac{d}{dx}\left(\dfrac{\sin x}{1+\cos x}\right)=\dfrac{(1+\cos x)\cos x+\sin x\sin x}{(1+\cos x)^2}$	M1	Quotient Rule (or Product Rule from $(\sin x)(1+\cos x)^{-1}$)
		A1	correct unsimplified
	$=\dfrac{\cos x+\cos^2 x+\sin^2 x}{(1+\cos x)^2}$	B1	use of $\sin^2 x+\cos^2 x=1$ oe
	$=\dfrac{1+\cos x}{(1+\cos x)^2}$	A1	completion
(ii)	$\int_0^2\left(\dfrac{1}{1+\cos x}\right)dx=\left[\dfrac{\sin x}{1+\cos x}\right]_0^2$	M1	correct integrand
	awrt 1.56	A1	

Question	Answer	Mark	Part Marks
4 (i)	$\mathrm{p}(2)=0 \to 8+4a+2b-24=0$ $\to (4a+2b=16)$	B1	
	$\mathrm{p}(1)=-20 \to 1+a+b-24=-20$ $\to (a+b=3)$	B1	
		M1	solve *their* linear equations for a or b
	$a=5$ and $b=-2$	A1	
(ii)	$\mathrm{p}(x)=x^3+5x^2-2x-24$	M1	find quadratic factor
	$=(x-2)(x^2+7x+12)$	A1	correct quadratic factor soi
	$=(x-2)(x+3)(x+4)$	M1	factorise quadratic factor and write as product of 3 linear factors
	$\mathrm{p}(x)=0 \to x=2,\ -3,\ -4.$	A1	if 0 scored, **SC2** for roots only
5 (i)	$AB^2=\left(\sqrt{3}+1\right)^2+\left(\sqrt{3}-1\right)^2$ $-2\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)\cos 60$	M1	use cosine rule
	$=3+1+2\sqrt{3}+3+1-2\sqrt{3}-2$	A1	at least 7 terms
	$=6$	A1	correct completion AG
(ii)	$\dfrac{\sin A}{\sqrt{3}-1}=\dfrac{\sin 60}{\sqrt{6}}$	M1	sine rule (or cosine rule)
	$\sin A=\dfrac{\left(\sqrt{3}-1\right)\sin 60}{\sqrt{6}}=\dfrac{\sqrt{6}-\sqrt{2}}{4}$ oe or 0.259 or 0.2588…	A1	correct explicit expression for $\sin A$ AG
(iii)	Area $=\dfrac{1}{2}\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)\sin 60$	M1	correct substitution into $\dfrac{1}{2}ab\sin C$
	$=\dfrac{\sqrt{3}}{2}$	A1	
6 (i)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\sec^2 x$	B1	
	$x=\dfrac{\pi}{4} \to \dfrac{\mathrm{d}y}{\mathrm{d}x}=\sec^2\dfrac{\pi}{4}=2$	B1	evaluated
	$y=8$	B1	
	Equation of tangent $\dfrac{y-8}{x-\dfrac{\pi}{4}}=2$ $(4-2y=\pi-16,\ y=2x+6.429\ldots,$ $\dfrac{\pi}{4}=0.7853\ldots)$	B1	

Question	Answer	Mark	Part Marks
(ii)	$\sec^2 x = \tan x + 7$		
	$\tan^2 x - \tan x - 6 = 0$ oe	M1	use $\sec^2 x = 1 + \tan^2 x$ to obtain a 3 term quadratic in $\tan x$
	$(\tan x - 3)(\tan x + 2) = 0$		
	$\tan x = 3$ or $\tan x = -2$	M1	
	$x = 1.25, \quad 2.03$	A1A1	solve three term quadratic for $\tan x$ extras in range lose final **A1**
7 (i)	$r^2 + h^2 = (0.5h + 2)^2$ oe	M1	
	$r^2 = 0.25h^2 + 2h + 4 - h^2$		
	$r^2 = 2h + 4 - 0.75h^2$	A1	correct expansion and r^2 subject and completion www AG
(ii)	$V = \frac{1}{3}\pi r^2 h = \frac{\pi}{3}\left(2h^2 + 4h - 0.75h^3\right)$	B1	any correct form in terms of h only
	$\frac{dV}{dh} = \frac{\pi}{3}\left(4h + 4 - 2.25h^2\right)$	M1	differentiate V
		A1	correct differentiation
	$\frac{dv}{dh} = 0 \rightarrow 2.25h^2 - 4h - 4 = 0$	M1	equate to 0 and solve 3 term quadratic
	$h = 2.49$ only	A1	cao
(iii)	$\frac{d^2V}{dh^2} = \frac{\pi}{3}(4 - 4.5h)$ when $h = 2.49$	M1	differentiate *their* 3 term $\frac{dV}{dh}$ and substitute *their* h
	(–7.545…) < 0 so maximum	A1	draw correct conclusion www
8 (i)	$\cos TOA = \frac{6}{10} \rightarrow$	M1	any method
	$TOA = 0.927$	A1	
(ii)	area of major sector =		
	$\frac{1}{2}6^2\left(2\pi - 2 \times \textit{their}\,0.927\right)$ (= 79.7)	M2	or **M1** for $\frac{1}{2}$ $6^2(2 \times \textit{their}\ 0.927)$ **DM1** for $\pi \times 6^2 - \frac{1}{2}$ $6^2(2 \times \textit{their}\ 0.927)$
	area of half kite = $\frac{1}{2}(6)\sqrt{10^2 - 6^2}$ (=24)	M1	
	area of kite × 2 (=48)	DM1	any method
	complete correct plan	DM1	*their* major sector + *their* kite
	awrt 128	A1	
(iii)	arc length =		
	$6 \times (2\pi - 2 \times \textit{their}\,0.927) + 2 \times \sqrt{10^2 - 6^2}\ )$	M1	complete correct method
	awrt 42.6	A1	

Question	Answer	Mark	Part Marks
9 (i)	$p=4$	**B1**	
(ii)	$\tan\alpha = \pm\frac{1}{3}$ or ± 3 or 18.4° or 71.6° seen	**M1**	could use cos or sin
	108	**A1**	
(iii)	$\boldsymbol{r_A} = \begin{pmatrix}1\\5\end{pmatrix} + t\begin{pmatrix}\text{their } p\\-3\end{pmatrix}$	**B1**	
(iv)	$\boldsymbol{r_B} = \begin{pmatrix}q\\-15\end{pmatrix} + t\begin{pmatrix}3\\-1\end{pmatrix}$	**B1**	
(v)	$5-3t=-15-t$	**M1**	$\boldsymbol{r_A} = \boldsymbol{r_B}$ and equate y/**j** and solve for t
	$\rightarrow t=10$	**A1**	
(vi)	$\begin{pmatrix}41\\-25\end{pmatrix}$ only	**B1**	
(vii)	$q=11$ only	**B1**	
10 (i)	$\mathrm{fg}(x)=\ln(2\mathrm{e}^x+3)+2$	**B1**	isw
(ii)	$\mathrm{ff}(x)=\ln(\ln x+2)+2$	**B1**	isw
(iii)	$x=2\mathrm{e}^y+3$	**M1**	change x and y and make e^y the subject
	$\mathrm{e}^y=\frac{x-3}{2}$		
	$\mathrm{g}^{-1}(x)=\ln\left(\frac{x-3}{2}\right)$ oe	**A1**	
(iv)	e^2 or 7.39	**B1**	
(v)	$\mathrm{gf}(x)=2\mathrm{e}^{(\ln x+2)}+3=20$	**B1**	gf correct and equation set up correctly
	$2\mathrm{e}^{\ln x}\mathrm{e}^2+3=20$	**M1**	one law of indices/logs
	$2x\mathrm{e}^2=17$	**M1**	second law of indices/logs
	$x=\frac{17}{2\mathrm{e}^2}$ or 1.15	**A1**	
			www if 0 scored, **SC2** for 17.3…

Question	Answer	Mark	Part Marks
11 (i)	$\mathbf{A}^2 = \begin{pmatrix} 2 & q \\ p & 3 \end{pmatrix}\begin{pmatrix} 2 & q \\ p & 3 \end{pmatrix} = \begin{pmatrix} 4+pq & 2q+3q \\ 2p+3p & pq+9 \end{pmatrix}$	**B2,1,0**	–1 each error
	$\mathbf{A}^2 - 5\mathbf{A} = 2\mathbf{I} \rightarrow 4 + pq - 10 = 2$ or $9 + pq - 15 = 2$	**M1**	equate top left or bottom right elements
	$\rightarrow pq = 8$	**A1**	accept $p = \frac{8}{q}$, $q = \frac{8}{p}$
(ii)	$\det \mathbf{A} = 6 - pq$	**B1**	
	$6 - pq = -3p$ and solve	**M1**	*their* det $\mathbf{A} = -3p$ and use *their* $pq = k$ oe to solve for p or q
	$\rightarrow p = \frac{2}{3}$	**A1**	
	$q = 12$	**A1**	**FT** from *their* $pq = k$